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Object Space Volume Rendering

Object space volume rendering

In object space rendering methods, the main loop is not over the pixels but over the objects in 3-space.

In the case of direct volume rendering, "objects" can mean:

- layers of voxels: image compositing methods
  - 2D texture based
  - 3D texture based
- voxels: splatting methods
- cells: cell projection methods

Texture-based volume rendering

Volume rendering by 2D texture mapping:

- use planes parallel to base plane (front face of volume which is "most orthogonal" to view ray)
- draw textured rectangles, using bilinear interpolation filter
- render back-to-front, using α-blending for the α-compositing

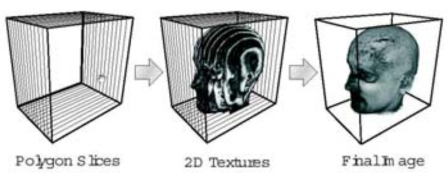


Image credit: H.W.Shen, Ohio State U.

Texture-based volume rendering

Volume rendering by 3D texture mapping (Cabral 1994):

- use the voxel data as the 3D texture
- render an arbitrary number of slices (eg. 100 or 1000) parallel to image plane (3- to 6-sided polygons)
- back-to-front compositing as in 2D texture method

Limited by size of texture memory.

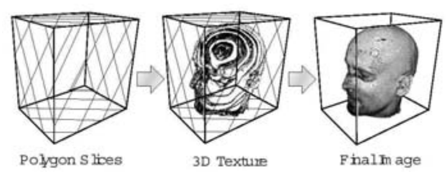


Image credit: H.W.Shen, Ohio State U.

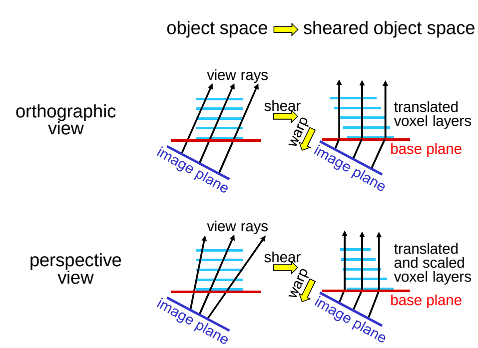
The shear-warp factorization

In general the image plane is not parallel to a volume face.

The shear-warp method by Lacroute allows to render an intermediate image in the base plane:

- transform to sheared object space by translating (and possibly scaling) the voxel layers
- render the intermediate image in the base plane
- warp the intermediate image

The shear-warp factorization



Orthographic shear-warp

The view transformation ("modelview" in OpenGL) is an affine transformation, consisting of a rotation and a translation.

Ignoring the translation, the 3x3 submatrix can be factorized:

$$\mathbf{M}_{view} = \mathbf{W} \cdot \mathbf{S} \cdot \mathbf{P}$$

where:

- **P** is a permutation matrix mapping the base plane (front face of the volume most orthogonal to the center view ray) to the xy-plane
- **S** is the shear matrix
- **W** is the warp matrix

Orthographic shear-warp

The shear is of the form

$$\mathbf{S}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + z\begin{pmatrix} s_x \\ s_y \\ 0 \end{pmatrix}$$

Hence, the shear matrix

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & s_x \\ 0 & 1 & s_y \\ 0 & 0 & 1 \end{bmatrix}$$

where s_x and s_y have to be solved for from $\mathbf{M}_{view}$.

Orthographic shear-warp

The warp is a 3x3 matrix, but effectively an affine transformation of the xy-plane.

The third row of **W** is irrelevant while two zeros in the third column are required to make the warp independent of z:

$$\mathbf{W} = \begin{bmatrix} w_{00} & w_{01} & 0 \\ w_{10} & w_{11} & 0 \\ w_{20} & w_{21} & w_{22} \end{bmatrix}$$

Orthographic shear-warp

Assuming for simplicity that **P** is the identity, we get:

$$\mathbf{M}_{view} = \begin{bmatrix} v_{00} & v_{01} & v_{02} \\ v_{10} & v_{11} & v_{12} \\ v_{20} & v_{21} & v_{22} \end{bmatrix} = \mathbf{W} \cdot \mathbf{S} = \begin{bmatrix} w_{00} & w_{01} & s_x w_{00} + s_y w_{01} \\ w_{10} & w_{11} & s_x w_{10} + s_y w_{11} \\ w_{20} & w_{21} & s_x w_{20} + s_y w_{21} + w_{22} \end{bmatrix}$$

It follows for the warp coefficients $w_{ij} = v_{ij} \quad (j \neq 2)$

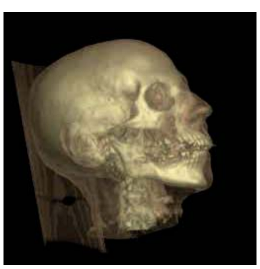
for the shear coefficients $\begin{pmatrix} s_x \\ s_y \end{pmatrix} = \begin{bmatrix} v_{00} & v_{01} \\ v_{10} & v_{11} \end{bmatrix}^{-1} \begin{pmatrix} v_{02} \\ v_{12} \end{pmatrix}$

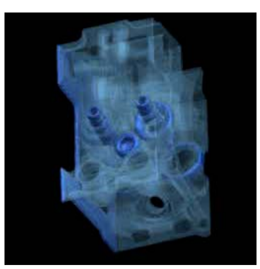
and for w_{22} (not needed) $w_{22} = -s_x v_{20} - s_y v_{21} + v_{22}$

If **P** is not the identity, permuted versions of **S** and **W** can be used.

Orthographic shear-warp

Example renderings: "VolPack" demos (P. Lacroute, Stanford U.)





Perspective shear-warp

The same factorization can be used, but now in homogenous coordinates:

$$\mathbf{M}_{view} = \mathbf{W} \cdot \mathbf{S} \cdot \mathbf{P}$$

The shear and scaling matrix S gets the form

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & s_x & 0 \\ 0 & 1 & s_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & s_w & 1 \end{bmatrix}$$

It does

- a translation of x by $s_x z$ and of y by $s_y z$, followed by
- a scaling with $1 / (1 + s_w z)$

The warp matrix **W** is:

$$\mathbf{W} = \begin{bmatrix} w_{00} & w_{01} & 0 & w_{03} \\ w_{10} & w_{11} & 0 & w_{13} \\ w_{20} & w_{21} & w_{22} & w_{23} \\ w_{30} & w_{31} & 0 & w_{33} \end{bmatrix}$$

The zero in the bottom row is needed to make the warp independent of *z*.

Assuming again that **P** is the identity, we get:

$$\mathbf{M}_{view} = \begin{bmatrix} v_{00} & v_{01} & v_{02} & v_{03} \\ v_{10} & v_{11} & v_{12} & v_{13} \\ v_{20} & v_{21} & v_{22} & v_{23} \\ v_{30} & v_{31} & v_{32} & v_{33} \end{bmatrix} = \mathbf{W} \cdot \mathbf{S} =$$

$$= \begin{bmatrix} w_{00} & w_{01} & s_x w_{00} + s_y w_{01} + s_w w_{03} & w_{03} \\ w_{10} & w_{11} & s_x w_{10} + s_y w_{11} + s_w w_{13} & w_{13} \\ w_{20} & w_{21} & s_x w_{20} + s_y w_{21} + w_{22} + s_w w_{23} & w_{23} \\ w_{30} & w_{31} & s_x w_{30} + s_y w_{31} + s_w w_{33} & w_{33} \end{bmatrix}$$

It follows for the warp coefficients $w_{ij} = v_{ij} \quad (j \neq 2)$

for the shear coefficients

$$\begin{pmatrix} s_x \\ s_y \\ s_w \end{pmatrix} = \begin{bmatrix} v_{00} & v_{01} & v_{03} \\ v_{10} & v_{11} & v_{13} \\ v_{30} & v_{31} & v_{33} \end{bmatrix}^{-1} \begin{pmatrix} v_{02} \\ v_{12} \\ v_{32} \end{pmatrix}$$

and for w_{22} (not needed) $w_{22} = -s_x v_{20} - s_y v_{21} - s_w v_{23} + v_{22}$

The shear-warp volume rendering algorithm is now as follows:

- For each voxel layer (parallel to base plane):
  - shear and scale the layer image by multiplying with **S**
  - apply transfer functions
- Generate intermediate image with α-compositing
- warp the image by multiplying with **W**

An advantage of this algorithm is that for scaling images a filter can be used to prevent undersampling (aliasing).

Object space vs. image space

Comparison of typical object space method (2D texture based) and image space method (raycasting).

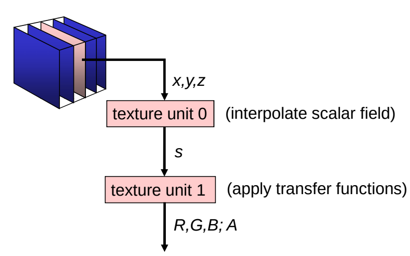
Formally both are equivalent, only different nesting order of loops.

Practical differences:

- Image space methods with FTB compositing allow early termination.
- Object space methods using framebuffer for intermediate results suffer from quantization artifacts.
- Object space methods can exploit texture mapping hardware and MIPmap textures for antialising.
- Image space methods would need supersampling in x and y for this.

Post-classification can be done in graphics hardware:

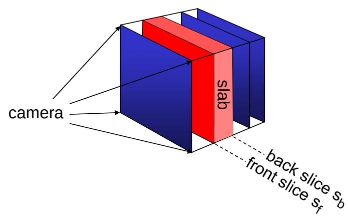
Using (OpenGL) dependent texture (two texture mapping stages):



Object space vs. image space

Preintegration is possible also in object space:

- Use slabs (space between two slices) instead of slices
- Dependent textures:
  - 1st stage: interpolate scalar field in front and back slice
  - 2nd stage: look up integrated transfer function



Splatting

Raycasting: "What does each voxel contribute to a given pixel?"

Splatting: "What does a given voxel contribute to each pixel?"

Splatting as a brute-force method:

- pre-processing:
  - for each voxel $\mathbf{x}_i$ render (raycast) a field $s(x_j) = \delta_{ij}$
  - store resulting footprint images
- main loop:
  - for each voxel $\mathbf{x}_i$ adjust footprint image to effective TF value
  - blend all footprint images of a voxel layer ("sheet buffer")
  - do α-compositing of layers

Splatting

Advantages of splatting:

- applicable to structured and unstructured grids
- other reconstruction filters than trilinear interpolation are possible, e.g. sinc filter

Original algorithm (Westover 1990):

- orthographic view, uniform grids $\rightarrow$ all footprints are translates of a template

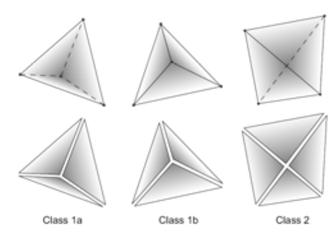
Elliptical weighted average (EWA) splatting (Zwicker et al. 2001)

- ellipsoidal Gaussians as footprints
- perspective view, low-pass filter for antialiasing

Cell projection

Projected tetrahedra (PT) is an object space method for tetrahedral grids [Shirley, Tuchman 1990].

Each (tetrahedral) cell is decomposed into 3 or 4 tetrahedra along those edges which are not part of the silhouette.



Cell projection

Cells are projected to triangle fans consisting of

- 1 thick vertex (projection of the common edge of the tetrahedra)
- 3 or 4 thin vertices (on the silhouette)

Original algorithm: triangle fan in the image plane

Improved algorithm: triangle fan in space:

- thin vertices keep original position
- thick vertex is set to midpoint of projected edge

Advantages:

- depth test can be used (allows volume rendering into a scene)
- viewing direction and field-of-view can be changed (for fixed camera position), keeping projection

Cell projection

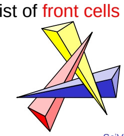
Computation of thick vertex:

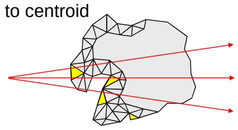
- compute determinants $d_i = \det(\mathbf{x}_j, \mathbf{x}_k, \mathbf{x}_l) \quad (i = 0,1,2,3)$
  where $\mathbf{x}_j, \mathbf{x}_k, \mathbf{x}_l$ are the vertices of the i^{th} face, relative to camera position, ordered ccw on outside of face
- if number of positive determinants is
  - odd: class 1
  - even: class 2
- interpolation weights (for coordinates and data) of thick vertex
  - for class 1: (example + + - +) $\frac{d_0}{2(d_0+d_1+d_3)}, \frac{d_1}{2(d_0+d_1+d_3)}, \frac{1}{2}, \frac{d_3}{2(d_0+d_1+d_3)}$
  - for class 2: (example - + + -) $\frac{d_0}{2(d_0+d_3)}, \frac{d_1}{2(d_1+d_2)}, \frac{d_2}{2(d_1+d_2)}, \frac{d_3}{2(d_0+d_3)}$

Assigning opacities:
- 0 for thin vertices
- preintegrated TF for thick vertex

Assigning colors:
- look up color TF for thin and thick vertices

Visibility sorting:
- generate partial ordering of cells based on adjacent pairs
- break cycles (rare, small rendering error, alternative: split a cell)
- sort list of front cells by distance to centroid





Rendering of triangles with fragment program:
- interpolate $s(\mathbf{x})$ for points on front and back triangle
- interpolate cell thickness
- lookup color and opacity in preintegrated TF

Back-to-front compositing
- cells must be depth-sorted
- possible without re-sorting: camera turn, zoom
- depth test (z-buffer) must be enabled
- additional (opaque) objects must be rendered before the volume

Example: Visualization of smoke propagation.
Simple smoke model (used in fire protection engineering):
- absorption τ proportional to $s(\mathbf{x})$ (particle concentration)
- leading to simple preintegrated(!) opacity TF: $\alpha = 1 - e^{-c\frac{\tau_f+\tau_b}{2}\|x_b - x_f\|}$

When compositing cells with low opacity, opacities are essentially added.

Adding many very small opacities (e.g. between 0/255 and 1/255) leads to quantization artifacts.

Options to reduce artifacts:
- compositing with 16 bits
- α–dithering: instead of standard rounding

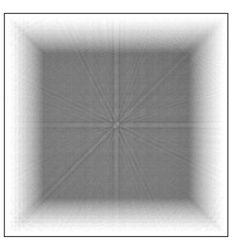
$$x \to \lfloor x \rfloor + \left(x - \lfloor x \rfloor \geq \frac{1}{2} \right)$$

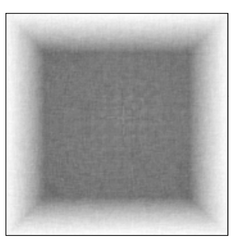
use randomized rounding

$$x \to \lfloor x \rfloor + \left(x - \lfloor x \rfloor \geq \text{rand} \right)$$

(Predicates $\geq$ understood as functions with values 0 and 1, 'rand' being a random function with range [0,1])

Example: Quantization artifacts without and with α-dithering.





Hardware-assisted visibility sorting (HAVS, Silva et al. 2005) is a faster cell projection algorithm:
- requires 4 RGBA float buffers for storing per pixel 7 pairs of
  - scalar field value *s*
  - distance *d* to camera
- initial cell sorting done by CPU, based on centroids, results in *k*-nearly sorted sequence, with $k \leq 7$
- main loop: draw all cell faces from back to front
- fragment shader
  - does exact sorting of buffered (*s*, *d*) pairs
  - computes "thickness" of cell behind the pixel, $\Delta d = d_1 - d_2$
  - does (preintegrated) TF lookup with $s_1, s_2, \Delta d$ and α-compositing