



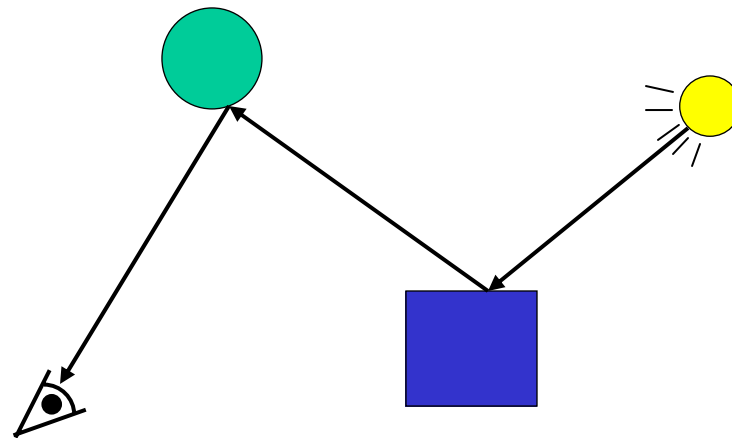
Lighting & Shading





Light Transport

- Goal: Model the interaction of light with matter in a way that appears realistic (and is fast)





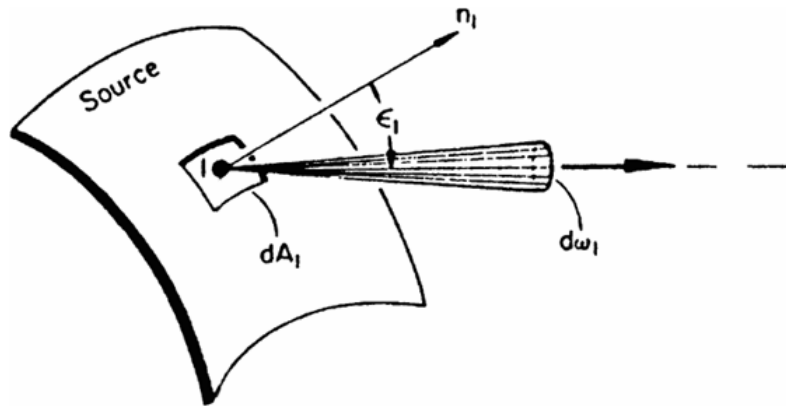
Light Transport

- Assumptions:
 - Geometrical optics:
 - No diffraction, no polarization, no interference
 - Light travels in a straight line in a vacuum
 - No atmospheric scattering or refraction
 - No gravity effects
 - Discrete-wavelength approximation of color
 - Quantized approximation of dispersion and fluorescence
 - Superposition
 - No non-linear reflecting materials



Bidirectional Reflectance

- Radiance L [W/(sr m²)]
 - radiant flux / (unit solid angle * unit area)



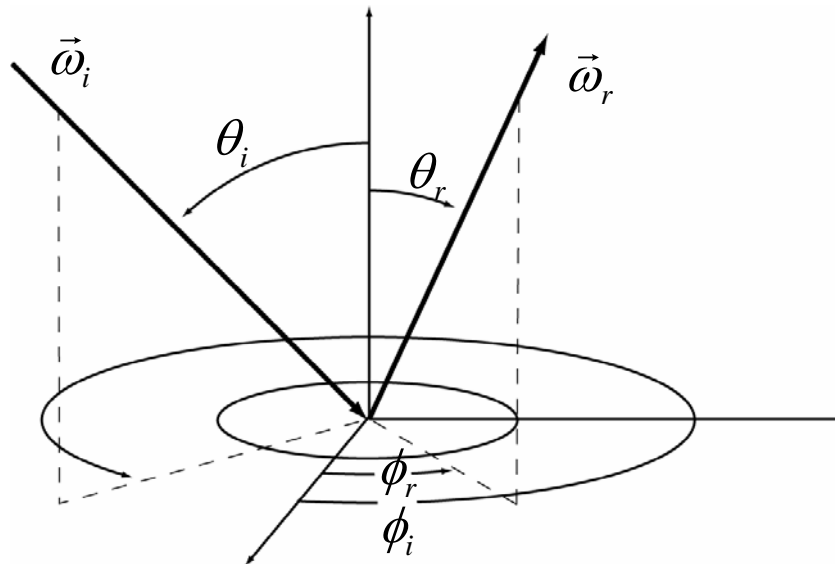


Bidirectional Reflectance

- Single incoming direction and single outgoing direction

$$L_r(\vec{\omega}_i, \vec{\omega}_r) = f_r(\vec{\omega}_i, \vec{\omega}_r) L_i(\vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

bidirectional reflectance distribution function



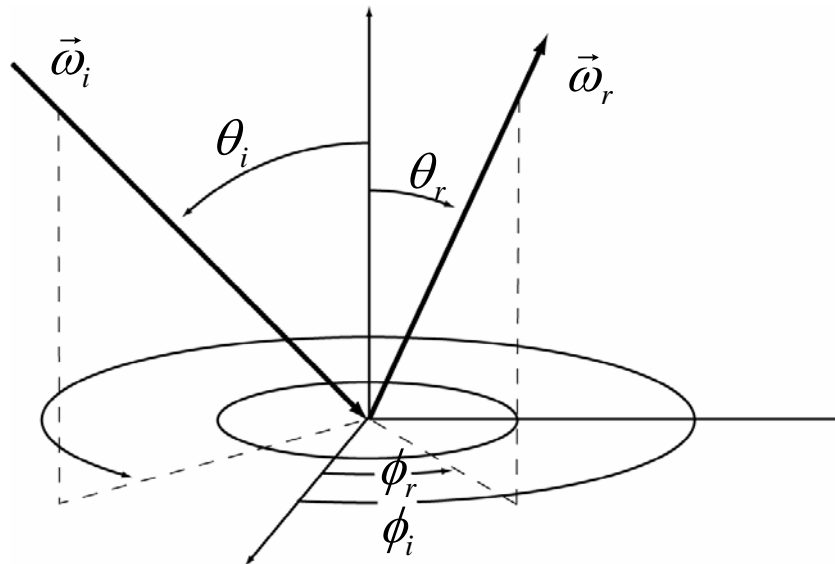


Bidirectional Reflectance

- Integral over all incoming directions

$$L_r(\vec{\omega}_r) = \int_{\Omega_i} f_r(\vec{\omega}_i, \vec{\omega}_r) L_i(\vec{\omega}_i) \cos \theta_i d\vec{\omega}_i$$

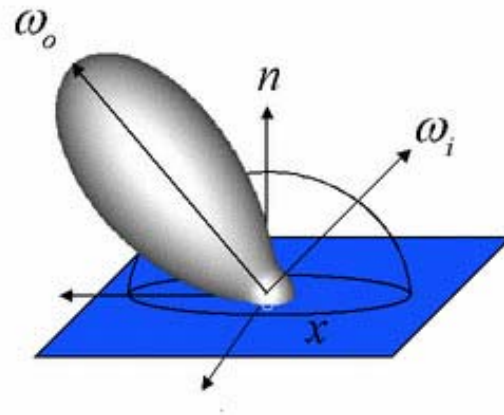
Ω_i ← incident hemisphere





Bidirectional Reflectance

- Bidirectional Reflectance Distribution Function (BRDF)
 - anisotropic (4 DOFs) $f_r(\phi_i, \theta_i, \phi_r, \theta_r)$
 - isotropic (3DOFs) $f_r(\phi_i - \phi_r, \theta_i, \theta_r)$





Bidirectional Reflectance

- Generalizations
 - anisotropic BRDF: 4 DOFs
 - spatially varying: +2
 - spectrally varying: +1
 - Fluorescence: +1
 - Phosphorescence: +1
 - Subsurface Scattering: +2
- General BRDF is 11 dimensional function!!



Subsurface Scattering



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Global Illumination



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Complex Materials and Lighting



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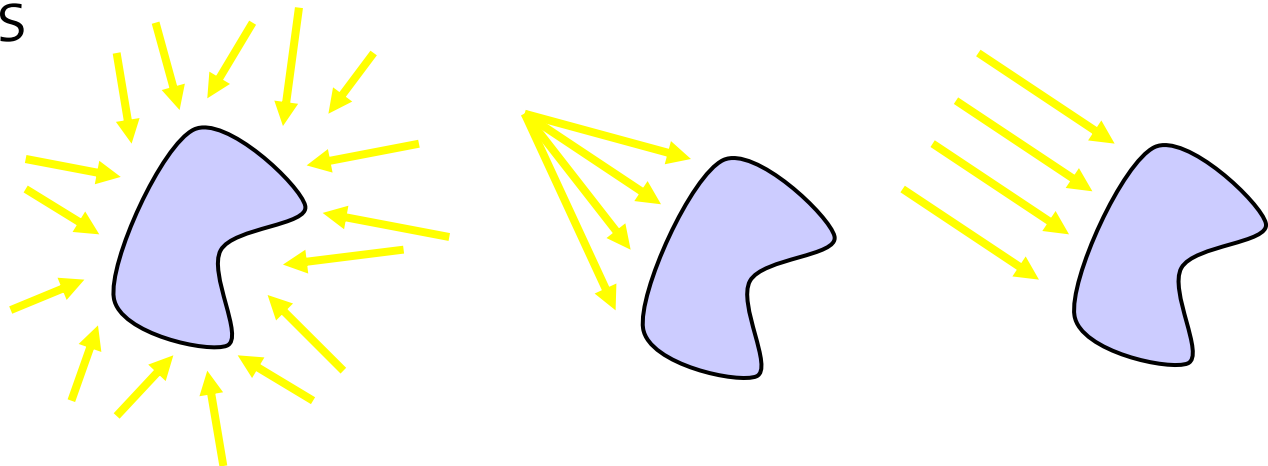
Real-World vs. OpenGL

- Real world
 - complex computations
 - see optics textbooks, photorealistic rendering
- OpenGL
 - simplified model
 - ambient, diffuse and specular light sources and reflections
 - easy to tune
 - fast to compute

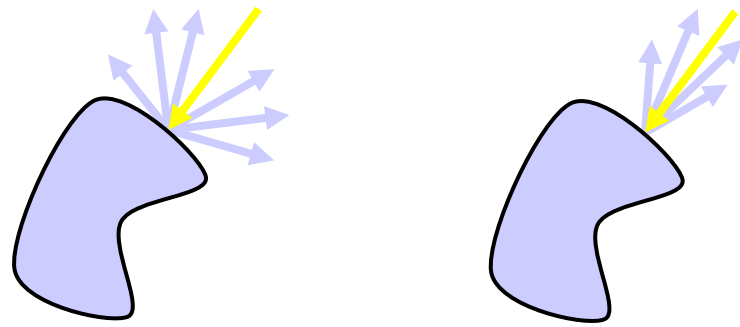


Lighting

- Light sources



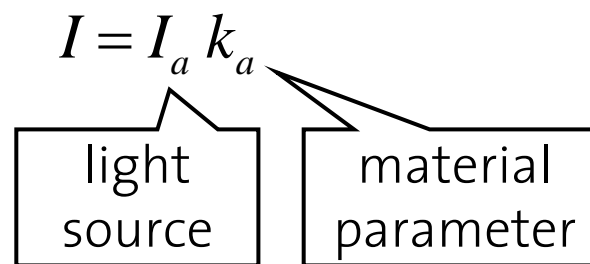
- Light reflection





Ambient Light

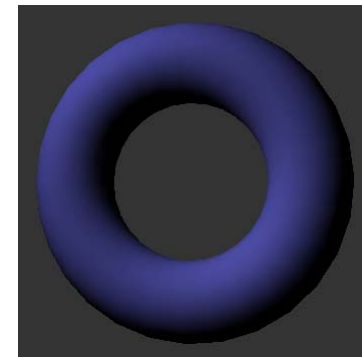
- Scattered by environment
- Coming from all directions
- Reflection **independent of**
 - Camera position
 - Light position (no light position)
 - Surface orientation
- Reflected intensity: $I = I_a k_a$



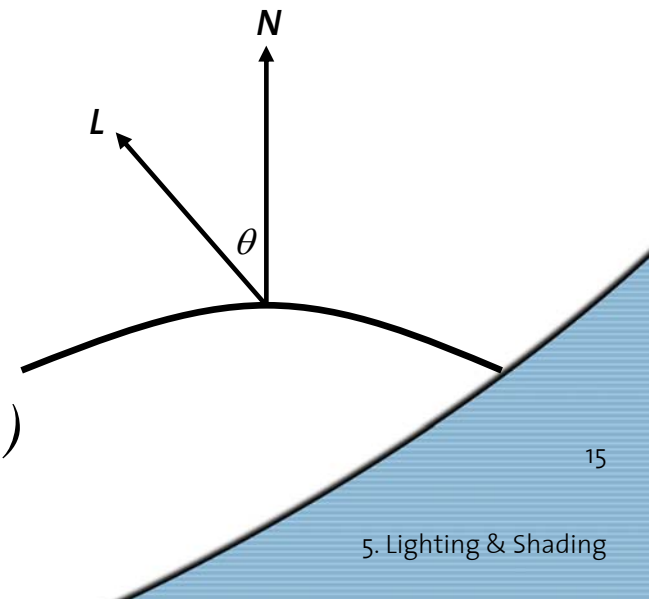


Diffuse Reflection

- Directed light I_p
- Reflection dependent on
 - orientation of surface
 - light source position
- Independent of
 - camera position
(reflected equally in all directions)
- Reflected intensity: $I = I_p k_d \cos \theta$



$$I = I_p k_d (\mathbf{N} \cdot \mathbf{L})$$

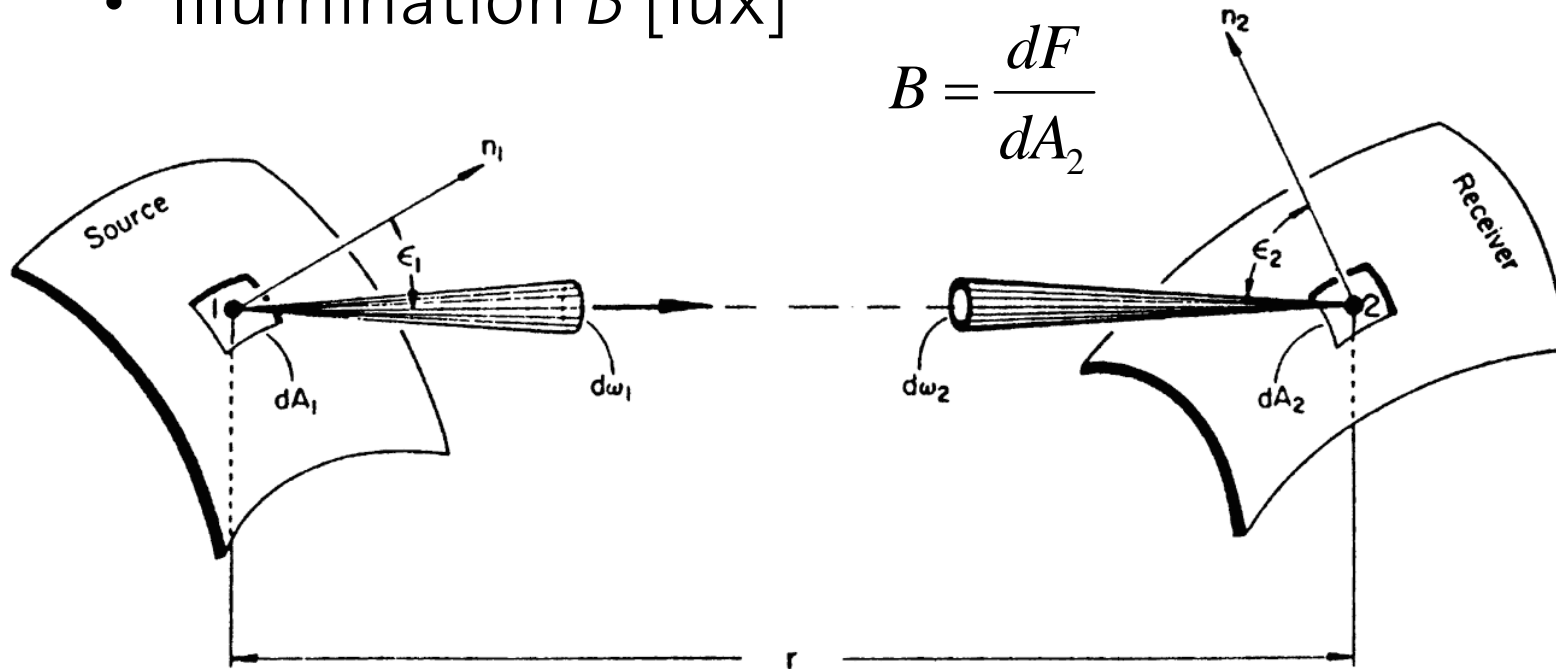




Two Radiant Surface Patches

- Illumination B [lux]

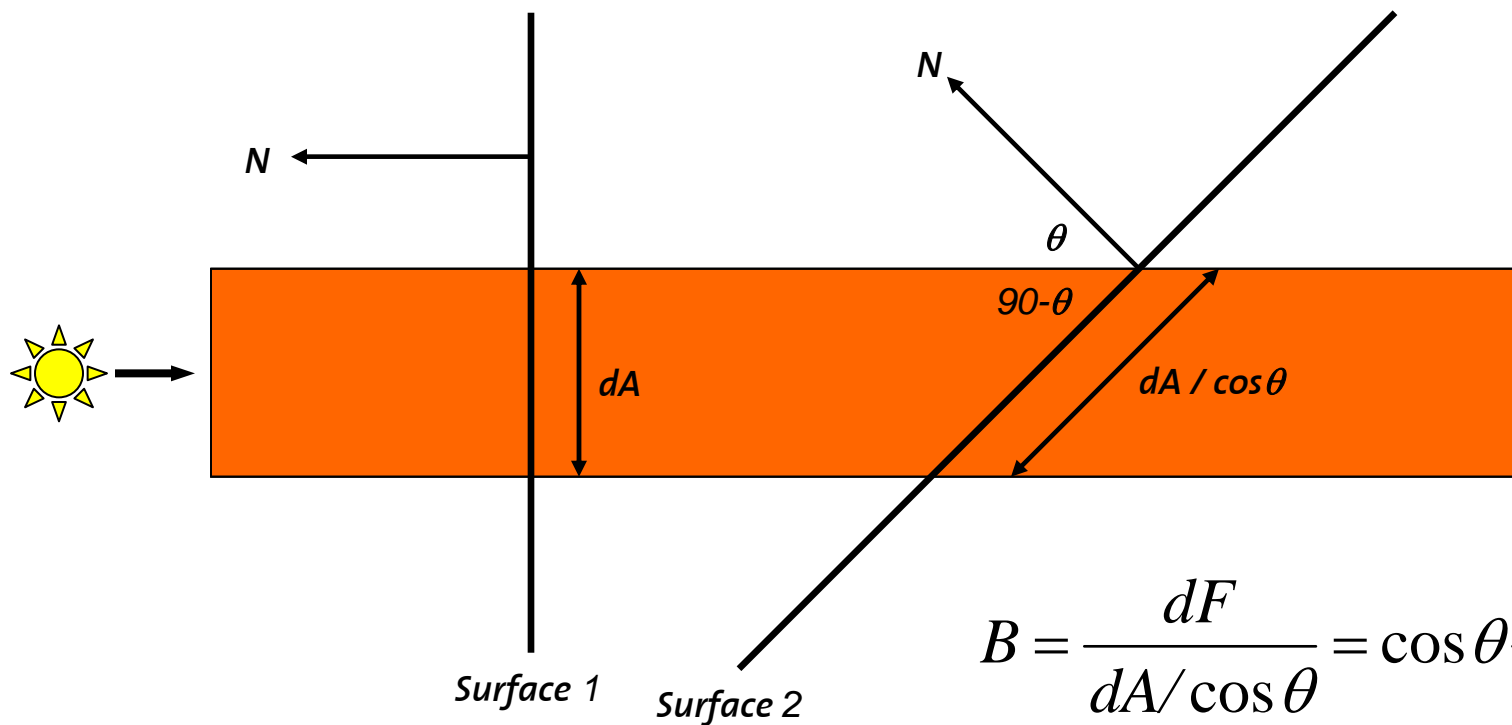
$$B = \frac{dF}{dA_2}$$





Diffuse Reflection

- Diffuse reflection scales with angle



$$B = \frac{dF}{dA / \cos \theta} = \cos \theta \frac{dF}{dA}$$

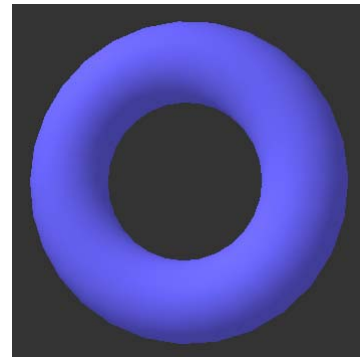
illumination



A Simple Model

- Sum up ambient light and diffuse reflection:

$$I = I_a k_a + I_p k_d (\mathbf{N} \cdot \mathbf{L})$$





Attenuation

- Quadratic attenuation due to spatial radiation

$$f_{att} = \frac{1}{d_L^2}$$

- A model often used in Graphics (OpenGL)

$$f_{att} = \min\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

```
glLightf(GL_LIGHT0, GL_CONSTANT_ATTENUATION, c1);  
glLightf(GL_LIGHT0, GL_LINEAR_ATTENUATION, c2);  
glLightf(GL_LIGHT0, GL_QUADRATIC_ATTENUATION, c3);
```

- Include attenuation

$$I = I_a k_a + f_{att} I_p k_d (\mathbf{N} \cdot \mathbf{L})$$



Wavelength Dependency

- Colored light and reflection as functions of wavelength λ

$$I_{\lambda} = I_{a_{\lambda}} k_a O_{d_{\lambda}} + f_{att} I_{p_{\lambda}} k_d O_{d_{\lambda}} (\mathbf{N} \cdot \mathbf{L})$$

- Restriction to RGB (OpenGL)

$$I_R = I_{a_R} k_{a_R} + f_{att} I_{p_R} k_{d_R} (\mathbf{N} \cdot \mathbf{L})$$

$$I_G = I_{a_G} k_{a_R} + f_{att} I_{p_G} k_{d_G} (\mathbf{N} \cdot \mathbf{L})$$

$$I_B = I_{a_B} k_{a_R} + f_{att} I_{p_B} k_{d_B} (\mathbf{N} \cdot \mathbf{L})$$



Restriction of spectral sampling to RGB can lead to substantial color artifacts



Depth Cueing (Fog)

- Linear depth cue by blending with the color of the participating medium

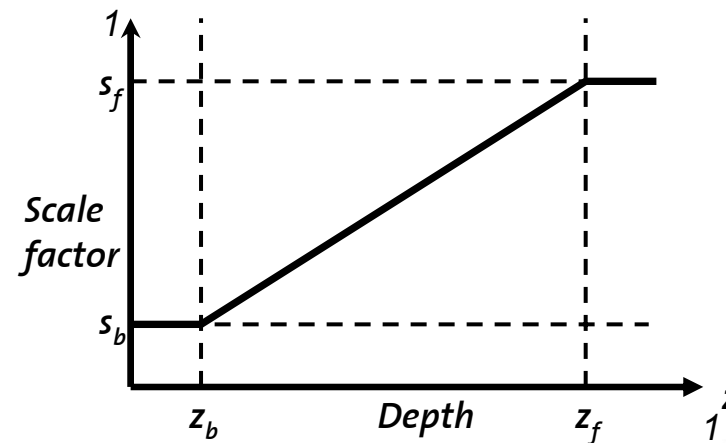
$$I_{\lambda} = sI_{\lambda} + (1-s)I_{dc_{\lambda}}$$



- s is scaling factor

$$s = s_b + \frac{(z - z_b)(s_f - s_b)}{z_f - z_b}$$

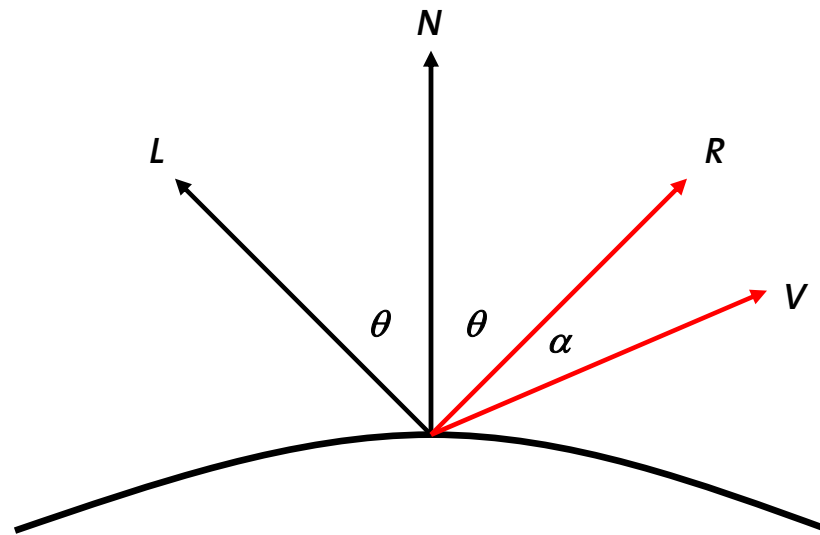
$$\text{for } z_b \leq z \leq z_f$$





Specular Reflection

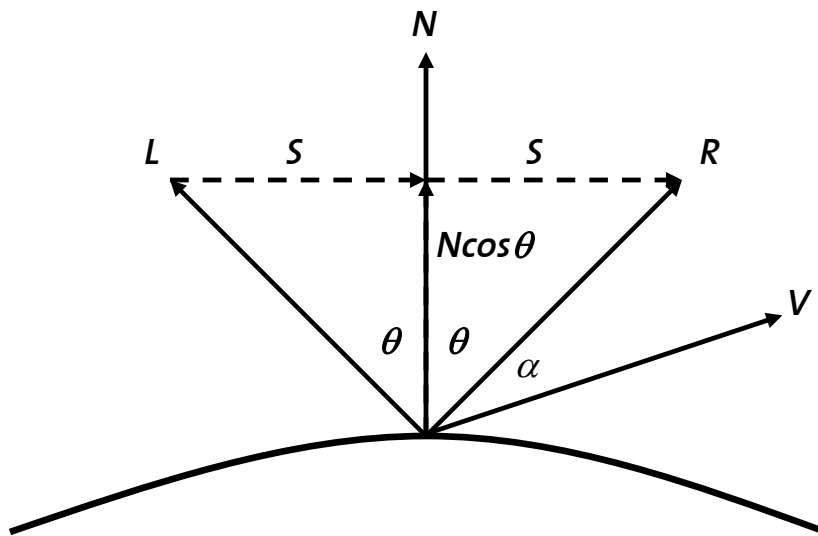
- Depends on angle between reflection and viewing ray





Reflected Ray

- Computed using simple vector algebra



$$\mathbf{R} = \mathbf{N} \cos \theta + \mathbf{S}$$

$$\mathbf{R} = 2\mathbf{N} \cos \theta - \mathbf{L} = 2\mathbf{N}(\mathbf{N} \cdot \mathbf{L}) - \mathbf{L}$$

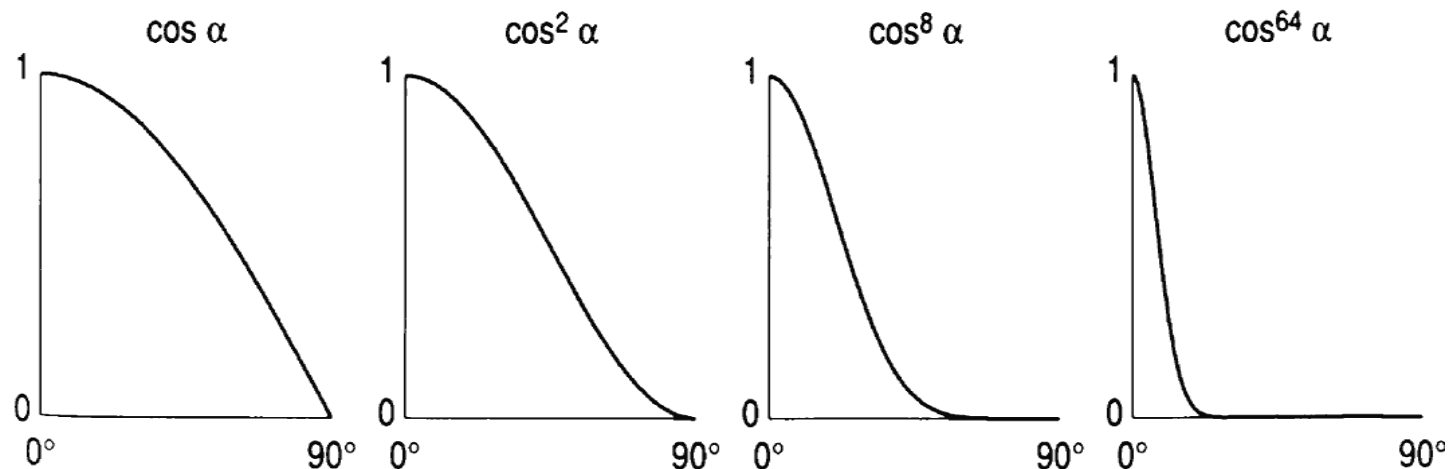
$$\cos \alpha = \mathbf{R} \cdot \mathbf{V} = (2\mathbf{N}(\mathbf{N} \cdot \mathbf{L}) - \mathbf{L}) \cdot \mathbf{V}$$



Phong Illumination Model

- Approximates specular reflection by cosine powers

$$I_{\lambda} = I_{a_{\lambda}} k_a O_{d_{\lambda}} + f_{att} I_{p_{\lambda}} [k_d O_{d_{\lambda}} (\mathbf{N} \cdot \mathbf{L}) + k_s (\mathbf{R} \cdot \mathbf{V})^n]$$

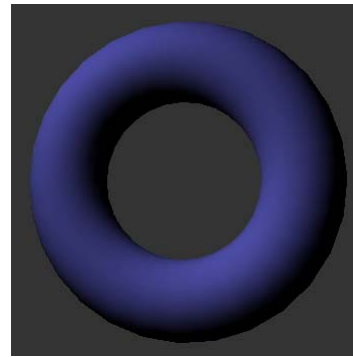




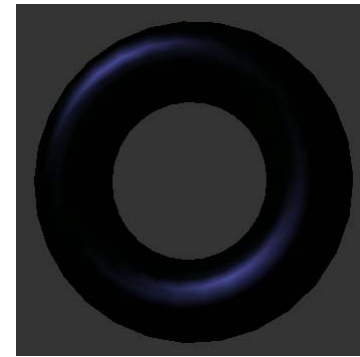
Ambient + Diffuse + Specular



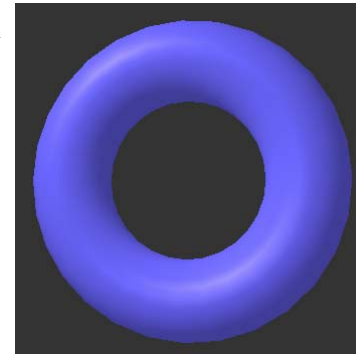
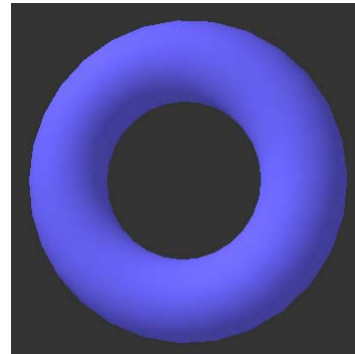
Ambient



Diffuse



Specular





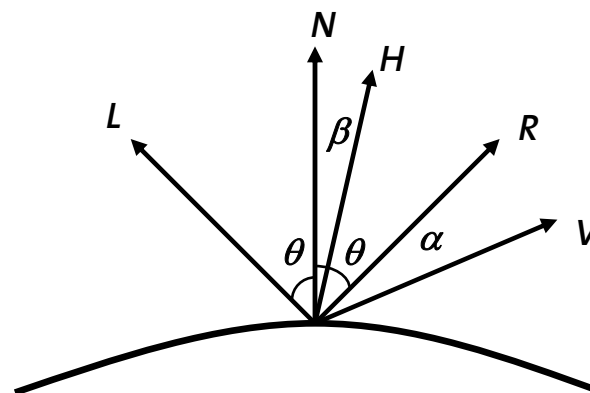
Extensions

- Specular colors

$$I_{\lambda} = I_{a_{\lambda}} k_a O_{d_{\lambda}} + f_{att} I_{p_{\lambda}} [k_d O_{d_{\lambda}} (\mathbf{N} \cdot \mathbf{L}) + k_s O_{s_{\lambda}} (\mathbf{R} \cdot \mathbf{V})^n]$$

- Halfway-Vector (faster)

$$\cos^n \beta = (\mathbf{N} \cdot \mathbf{H})^n \quad \mathbf{H} = \frac{\mathbf{L} + \mathbf{V}}{|\mathbf{L} + \mathbf{V}|}$$



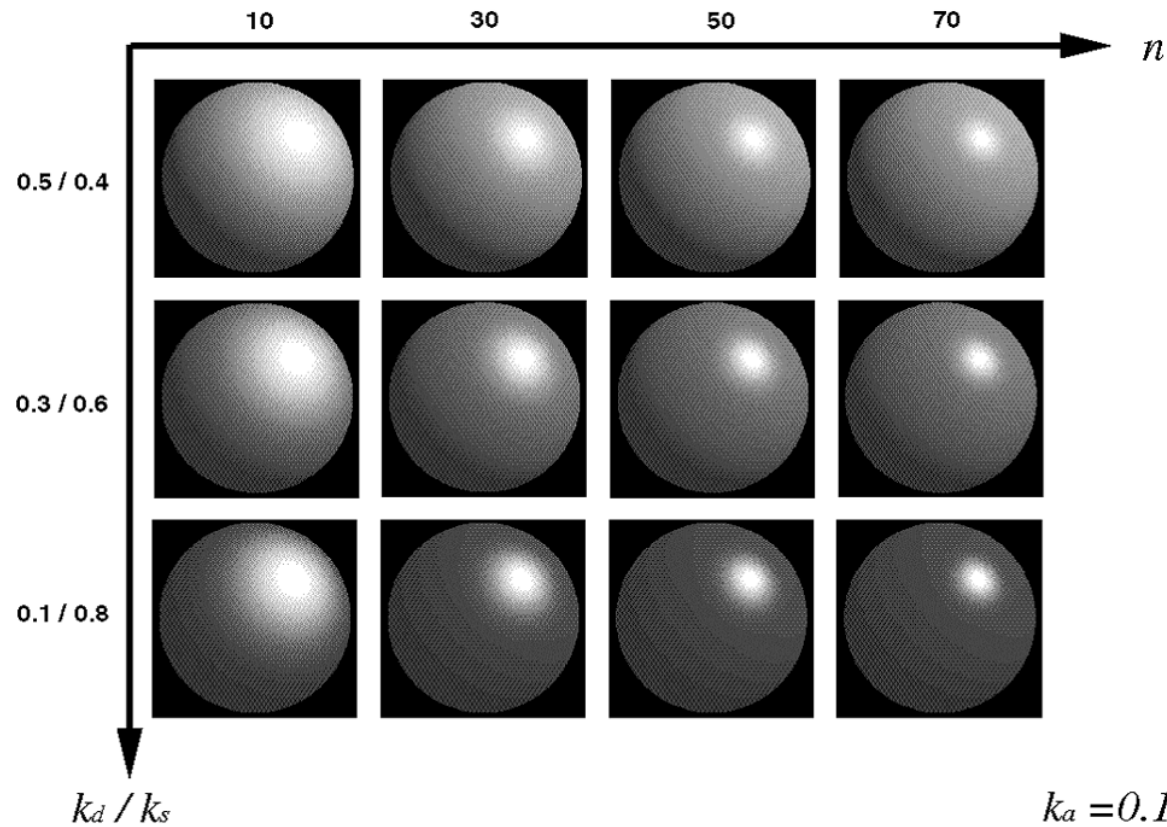
- Multiple Light Sources

$$I_{\lambda} = I_{a_{\lambda}} k_a O_{d_{\lambda}} + \sum_{1 \leq i \leq m} f_{att_i} I_{p_{\lambda i}} [k_d O_{d_{\lambda}} (\mathbf{N} \cdot \mathbf{L}_i) + k_s O_{s_{\lambda}} (\mathbf{R}_i \cdot \mathbf{V})^n]$$



Highlights

- k_s/k_d determines highlighted surface regions



$k_a = 0.1$



OpenGL

- Light source:

```
GLfloat light_pos[] = {x,y,z, 1};
GLfloat light_Ia[] = {IaR, IaG, IaB, IaA};
GLfloat light_Id[] = {IdR, IdG, IdB, IdA};
GLfloat light_Is[] = {IsR, IsG, IsB, IsA};
glLightfv(GL_LIGHT0, GL_POSITION, light_pos);
glLightfv(GL_LIGHT0, GL_AMBIENT, light_Ia);
glLightfv(GL_LIGHT0, GL_DIFFUSE, light_Id);
glLightfv(GL_LIGHT0, GL_SPECULAR, light_Is);
```

- Reflection
(material):

```
GLfloat material_Ka[] = {kaR, kaG, kaB, kaA};
GLfloat material_Kd[] = {kdR, kdG, kdB, kdA};
GLfloat material_Ks[] = {ksR, ksG, ksB, ksA};
glMaterialfv(GL_FRONT, GL_AMBIENT, material_Ka);
glMaterialfv(GL_FRONT, GL_DIFFUSE, material_Kd);
glMaterialfv(GL_FRONT, GL_SPECULAR, material_Ks);
glMaterialfv(GL_FRONT, GL_SHININESS, Se);
```



Light Position

- Position in world-coordinates
- Light stationary

```
gluLookAt(eyeX, eyeY, eyeZ,  
          centerX, centerY, centerZ,  
          upX, upY, upZ);  
glLightfv(GL_LIGHT0, GL_POSITION, pos);
```

- Position in camera-coordinates
- Light moving with camera (headlight)

```
glLightfv(GL_LIGHT0, GL_POSITION, pos);  
gluLookAt(eyeX, eyeY, eyeZ,  
          centerX, centerY, centerZ,  
          upX, upY, upZ);
```

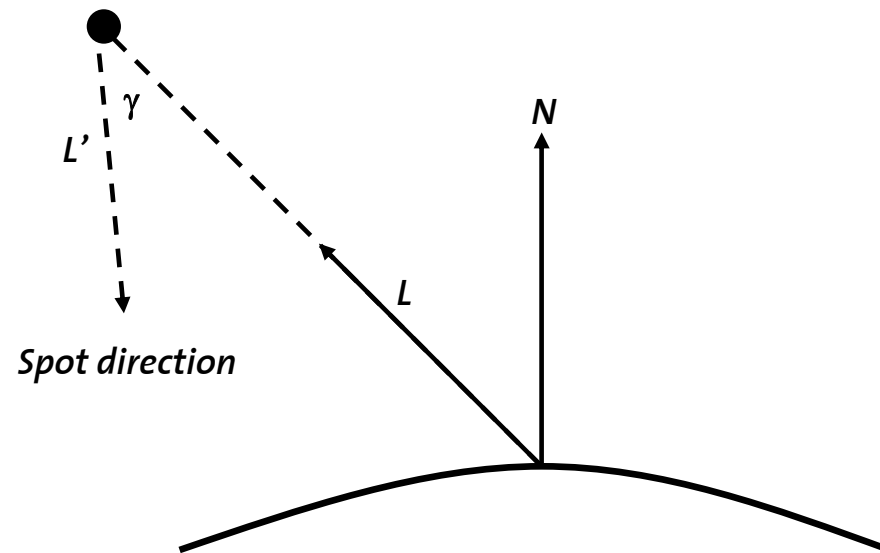




Light Sources

- Directed Spotlights

Spot position



$$I_{L'\lambda} \cos^p \lambda$$

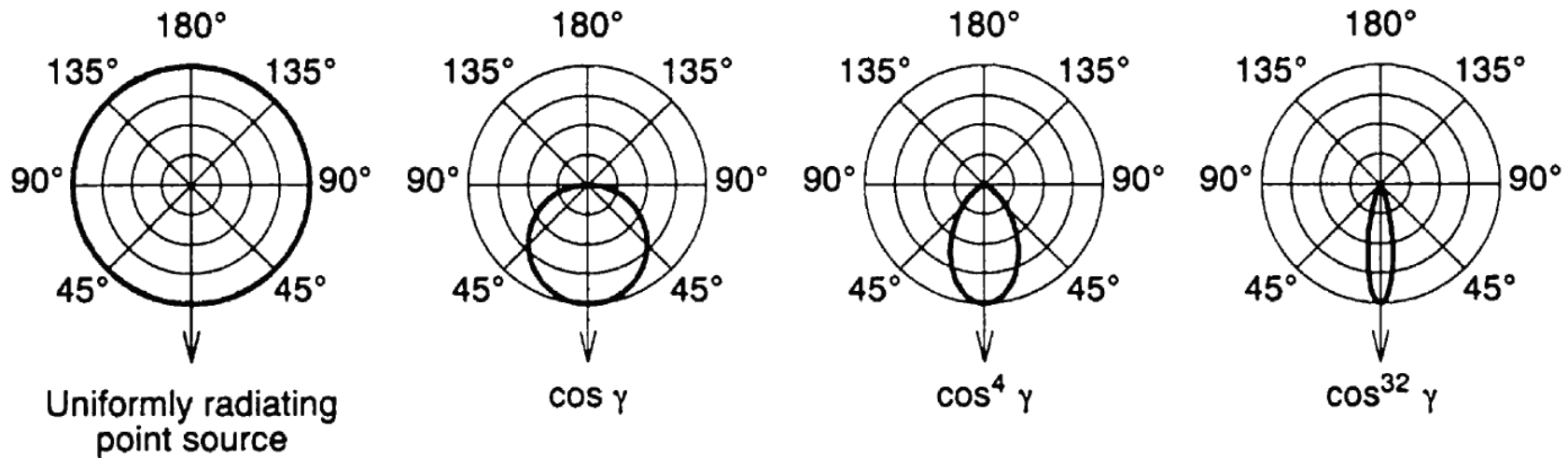
$$I_{L'\lambda} (-L \cdot L')^p$$

```
glLightfv(GL_LIGHT0, GL_SPOT_DIRECTION, L');  
glLightf(GL_LIGHT0, GL_SPOT_EXPONENT, p);
```



Intensity Diagrams

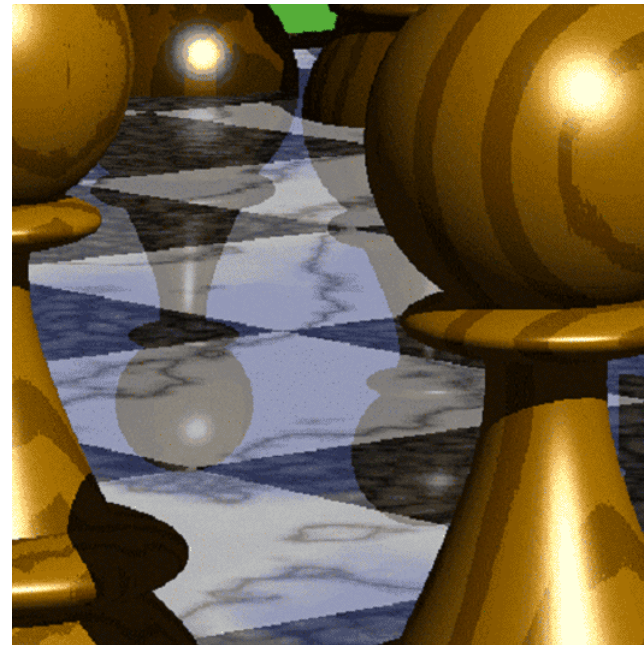
- Implemented in most modern graphics APIs





Shading

- Shading requires many evaluations of a lighting model
- Questions:
 - where to evaluate ?
 - when to evaluate ?





Shading Models

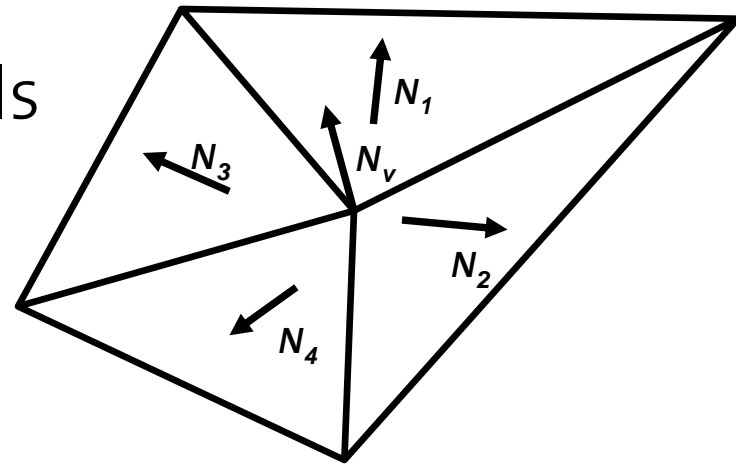
- ***Flat shading*** (constant shading)
 - one color per primitive
- ***Gouraud shading***
 - linear interpolation of vertex intensities
- ***Phong shading***
 - linear interpolation of vertex normals
- Gouraud & Phong shading need vertex normals



Gouraud Shading – Procedure

1. Calculate face normals
2. Calculate vertex normals

$$N_v = \frac{\sum_{i=1}^n N_i}{\left| \sum_{i=1}^n N_i \right|}$$

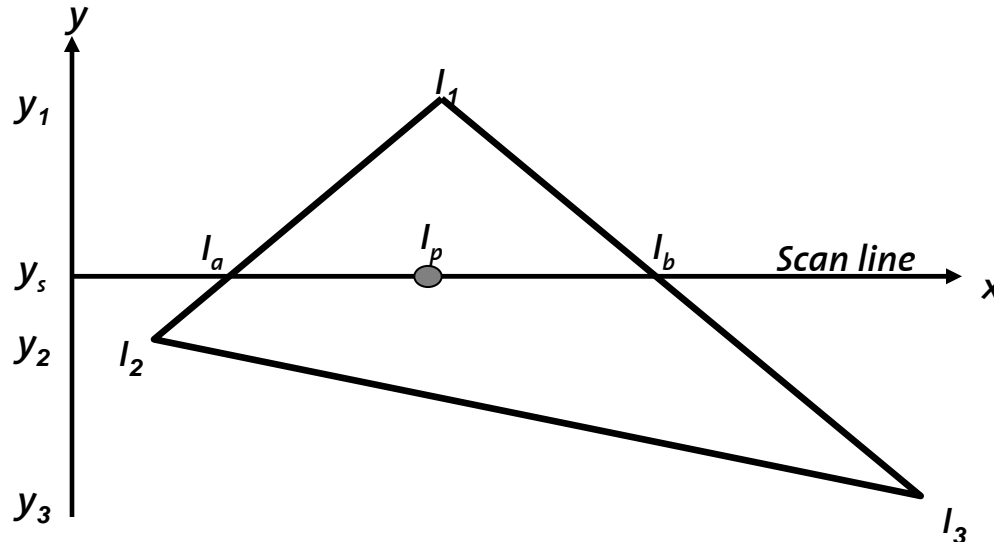


3. Evaluate lighting model for each vertex
4. Interpolate vertex intensities across primitive (bilinear interpolation)



Gouraud Shading – Interpolation

- Linear interpolation on current scan line



$$I_a = I_1 - (I_1 - I_2) \frac{(y_1 - y_s)}{(y_1 - y_2)}$$

$$I_b = I_1 - (I_1 - I_3) \frac{(y_1 - y_s)}{(y_1 - y_3)}$$

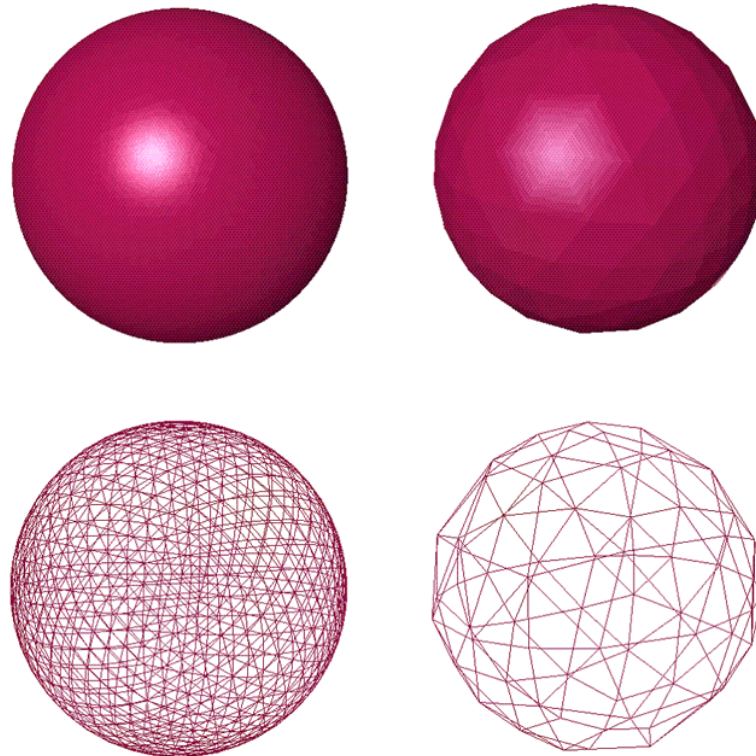
$$I_p = I_b - (I_b - I_a) \frac{(x_b - x_p)}{(x_b - x_a)}$$

- Done during scan conversion
- Hardware acceleration possible



Gouraud Shading – Quality

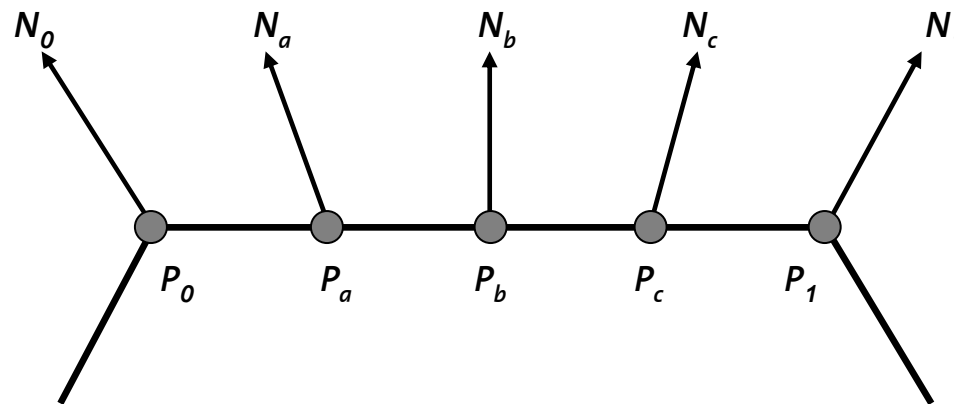
- Shading quality depends on size of projected primitives (relative to pixel size)





Phong Shading

- Linear interpolation of normals (not intensities) on current scan line



- Evaluation of lighting model at each pixel
- More accurate but slower than Gouraud



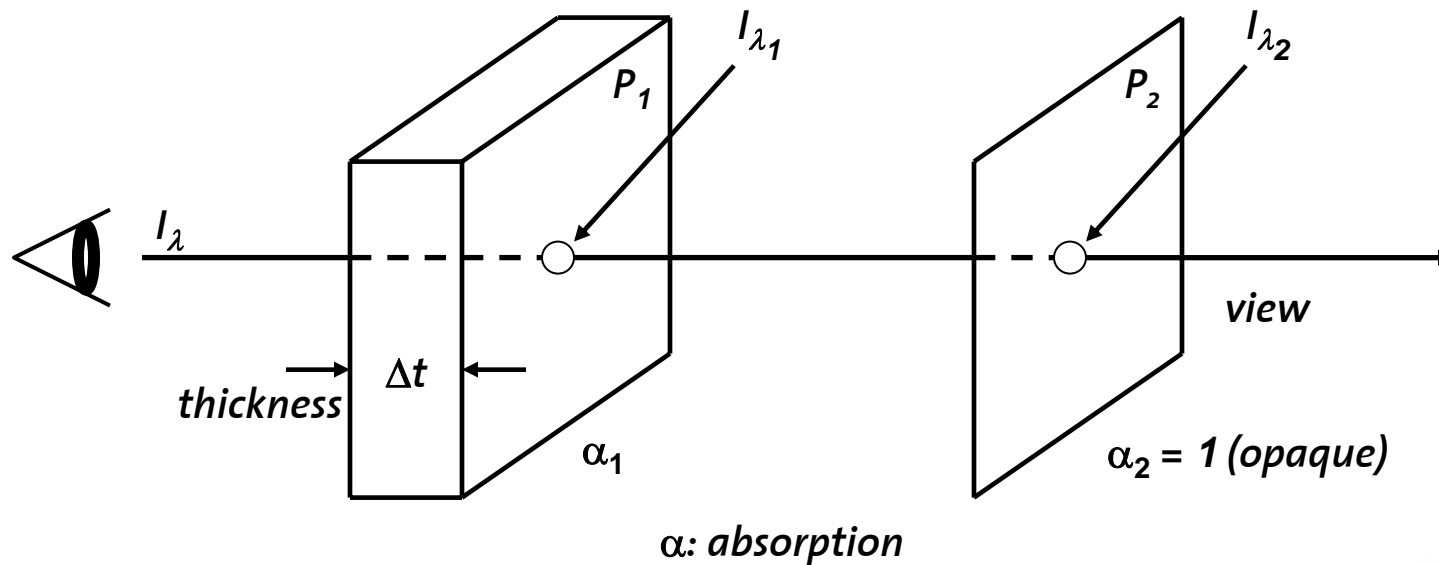
Shading Methods – Overview

- Shading in image space:
 - *Flat shading* (*Constant shading*)
 - *Gouraud shading*
- Shading in object / screen space:
 - *Phong shading*
- Impact on graphics hardware...



Transparency

- Transparency done with α -blending...
- Linearizing exponential attenuation of intensity in media





Transparency – Basic Laws

- P1 filters intensity according to $I'_{\lambda_2} = I_{\lambda_2} e^{-\alpha_1 \Delta t}$
- Linearization yields $I'_{\lambda_2} = I_{\lambda_2} (1 - \alpha_1 \Delta t)$
- Emission of P_1 $I'_{\lambda_1} = I_{\lambda_1} \alpha_1 \Delta t$
- Summing up (for unit length)
$$I_{\lambda} = I'_{\lambda_1} + I'_{\lambda_2} = I_{\lambda_1} \alpha_1 + I_{\lambda_2} (1 - \alpha_1)$$
- For N polygons (**α -blending**)

$$I_{\lambda} = \sum_{i=1}^N \alpha_i I_{\lambda_i} \cdot \prod_{b=1}^{i-1} (1 - \alpha_b)$$