Simulation of Morphology Changes in Drying Leaves

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Abstract
We introduce a biologically motivated simulation technique for the realistic shape deformation of drying leaves. In contrast to skeleton-based leaf deformation, our approach simulates the whole leaf surface to capture the fine details of desiccated leaves. We represent a leaf as a triangulated double-layer structure that consists of a Delaunay triangulation discretized along the vein structure and its corresponding Voronoi diagram. This structure can generate not only sharp creases along leaf veins, but also the complicated curling and crumpling on the leaf surface. The loss of water is the major factor that controls the inhomogeneous shrinkage of drying leaves. The proposed osmotic water flow successfully models the gradual changes of dehydrated regions advancing towards the veins. We demonstrate the robustness of our method by comparing a sequence of simulated morphology changes with photographs of real leaves.

Keywords: Osmotic water flow, triangle-Voronoi double-layer structure.

1. Introduction
Dying and drying leaves are an essential component of autumn scenes; but their morphology is complicated. They wrinkle, curl and crumple while generating folds along their leaf veins. The veins on a leaf are conspicuous and support the leaf surface. Therefore, existing methods of modeling and deforming leaves have been focused on veins [MMPP03, HSVGB05, RFL∗05, PTMG08]. The simple interpolations [MMPP03, HSVGB05] used in these methods can capture the curved shapes of desiccated leaves, but neglect the fine details found on the leaf surface (Fig. 17 (c)). This paper focuses on the morphological features and changes of the drying leaf surface based on biological theories.

A drying leaf generates wrinkles on the surface, because it buckles rather than compresses to minimize the stress induced by inhomogeneous shrinkage. We use a cloth simulation technique [MHHR07] to capture the fine wrinkles on a dehydrated leaf surface. Cloth is usually represented as a 2D manifold mesh, which requires an additional bending model to simulate buckling [CK02]. We model a leaf as a double layer of a Delaunay triangulation and a Voronoi diagram. This is a triangular analogy of a double-layer structure based on body-centered squares [JKK11], and it inherits the advantage that both bending and shrinkage can be controlled by the length of springs. The triangular version is more appropriate for the irregular mesh formed by leaf veins.

Water accounts for 70% of the volume of leaf cells, and so dehydration causes cell shrinkage [KB95]. This leads us to control the shrinkage of drying leaves according to the loss of water. Therefore, we simulate the internal water flow of a leaf and the shrinkage depending on the water change. Adjusting the diffusion coefficient or boundary conditions in the water flow model allows us to generate a variety of dehydration scenarios.

The boundary regions of a leaf shrink more: they cannot easily replenish lost water because of their distance from veins. Leaf water flows through the permeable membranes that separate the leaf cells because of the different concentrations of solutes in adjacent cells: this is called osmosis [KB95]. Living leaves have an inhomogeneous distribution
of water and solutes which causes water to diffuse from the veins to the boundary regions. However, diffusion models that force all regions to have the same water content in the equilibrium state cannot accomplish the inhomogeneous distribution of water in a leaf. In this paper, we simulate the osmotic water flow by extending an existing diffusion model [DMSB99]. The flow is controllable by designing the distribution of solutes, because water flows in the direction of the concentration gradient.

We present a simulation technique for the gradual shape changes of a single drying leaf focusing on the deformation quality by capturing detailed features. This paper makes the following contributions.

- We introduce the triangle-Voronoi double-layer structure to generate complicated wrinkles and folds by inhomogeneous shrinkage.
- Simulation of the osmotic diffusion of water flowing according to the concentration gradient allows users to control the distribution and flow of water along the leaf venation.

2. Related Work

Interactive techniques for modeling 3D curved leaves allow the user to manipulate the vein skeletons. The shape of the leaf surface can then be approximated using the deformed veins by free-form deformation [MMPP03] or harmonic interpolations [HSVGB05]. An L-system is a practical method of generating large numbers of self-similar plants by applying production rules recursively. The L-system based modeling technique [PMKL01] can generate curved leaves by rules that twist and bend the generalized cylinder sweeping the leaf along its axis. Peyrat and Galin [PTMG08] encode the rules for the evolution of a leaf’s shape and color in their 2Gmap L-system, and simulate aging by making the angles between veins parameters of a grammar.

Although the modeling techniques using vein skeletons succeeded in obtaining the overall curved shapes of leaves, detailed features such as curling or bending of the leaf surface cannot be achieved. This is because the degree of freedom of the leaf surface completely relies on the complexity of the skeleton.

In the biological view, the curled shape of the desiccated leaf is a buckling of the leaf surface caused by the non-uniform volume changes in local regions [CRM∗04]. Xiao and Chen [XC11] focus on the curled morphology observed in drying leaves. Using the experimentally measured strains of dried leaves, they create a linear differential strain field related to the distance from the boundary of an elliptical shell and approximate the normalized average curvature to make the leaf buckle by shrinkage. Varying the material properties of veins, such as thickness and elasticity, causes wavy features to become prominent. The resulting deformations are plausible, except that veins close to the stalk distorted unnecessarily because of their proximity to the edge. To resolve this problem, we approximate the dehydration rates of local regions depending on the distance from the closest veins; as a result, the regions away from the main vein wrinkle faster and to a greater extent (Fig. 1).

To the best of our knowledge, research on the deformation of drying leaves under non-uniform shrinkage or expansion has not been introduced in computer graphics. Instead, we have surveyed the simulation techniques of other natural phenomena. The simulation of flower opening [IYKI08] is similar to the growth of leaves in that flower petals bend because of the different expansion rates of their upper and lower surfaces.

The simulation controls the non-uniform expansion of the two surfaces by designed textures. Kider et al. [KJR11] use a mass-spring system to simulate the rotting of fruit, in which fungus spreads on the surface and the volume decreases by dehydration.
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3. Leaf Modeling

A leaf is a multicellular structure in which different types of cells are layered in turn (Fig 2). The mesophyll layer fills the internal leaf volume between the upper and lower epidermis, and cuticles covering the epidermis provide a waterproof leaf from being wet or dry. The effect of water on the organelles causes complicated volume changes. Shrinkage rates can be determined by experiment [XC11] or simply linked to water loss.

In order to obtain realistic wrinkling, we model leaves with a double-layer structure. We considered using body-centered squares, which have been used to model the complicated wrinkling of burning shells by shrinking the length of springs in proportion to mass loss [JKK11]. However, the regularity of the body-centered squares makes them unsuitable for modeling leaves, which have sharp creases along their veins. We have therefore developed a similar structure, but based on triangles.

3.1. Model Acquisition

The initial leaf model in our simulation consists of a naturally curved triangular double-layer structure that accommodates the leaf outline and vein structure.

Our modeling process starts with a photograph of a single leaf and an auxiliary image (Fig. 3 (a, b)) which identifies the vein structure and the silhouette of the leaf. We create vein particles (green points) and their connecting springs by sampling the auxiliary image, and generate a regular distribution of random particles (black points) to obtain an even distribution (Fig. 3 (c)). Then, we triangulate all these particles to produce a triangular mesh (Fig. 3 (d)).

It is straightforward to create a 3D double-layer structure from the triangular mesh (Fig. 3 (e)). Creases along veins and a curved leaf surface are achieved by adjusting the length of the springs on the Voronoi layer. We shrink the vein springs to make creases and expand the remaining springs to make the leaf curl using our deformation system (Fig. 3 (f)). In the example of Fig. 3, vein particles on the Voronoi layer are positioned in the opposite direction to the surface. Either the vein particles in the normal direction shrink or those in the opposite direction expand; both result in the same creases (Fig 4(b)).

3.2. The Triangle-Based Double-Layer Structure

Our double-layer structure consists of a Delaunay triangulation and its corresponding Voronoi diagram, we call these two triangle and Voronoi layers, and they are connected by a third ‘link layer’, which maintains the thickness between them.

This structure is shown in Fig. 5. A Voronoi particle $p_i^V$ is simply located at the center of each triangle, and a Voronoi spring $s_{ij}^V$ which links pairs of Voronoi particles in adjacent triangles (Fig. 5 (b)). Three link springs $s_{ij}^L$ connect the
Voronoi particle at the center of a triangle with the three triangle particles (Fig. 5 (c)). Thus, there are two types of particles $p^T$ and $p^V$ and three types of springs $s^T$, $s^V$ and $s^L$.

A particle $p_i$ in either of the particle sets $p^T$ and $p^V$ has a position $x_i$, a velocity $v_i$, a water content $w_i$, a solute content $M_i$, a total mass $m_i (= M_i + w_i)$. A spring $s_j$ in one of the three spring sets $s^T$, $s^V$ and $s^L$ connects two particles $p_a^j$ and $p_b^j$ and has the target length $l_j$.

Each triangle has a thickness $h_i$, which is the shortest distance between the plane of the triangle and its corresponding Voronoi particle (Fig. 5 (e)). If the thickness $h_i < 0$, this means that the corresponding Voronoi particle is positioned in the opposite direction to the face normal (Fig. 4). Given two simple models that have the same properties except thickness (Fig. 6), the thickness $h$ affects the bending angle $\Delta \theta$ between two adjacent cells when the upper layer shrinks at the same rate $\Delta l$. A thinner model wrinkles more, because $\Delta \theta$ is inversely related to $h$: $\Delta \theta = \tan^{-1}(\Delta l/h)$ [IYKI08].

3.3. Mass Distribution w.r.t. the Vein Structure

As leaves dehydrate, veins become more conspicuous because they bend into smooth curves, whereas the leaf surface wrinkles. We therefore assign different properties to the vein particles $p_{\text{vein}}$ and vein springs $s_{\text{vein}}$ on the triangle and Voronoi layers. $p_{\text{vein}}^T$ and $s_{\text{vein}}^T$ on the triangle layer are chosen when the leaf is modeled, whereas the vein springs $s_{\text{vein}}^V$ on the Voronoi layer connect the Voronoi particles corresponding to the two adjacent triangles which share vein springs $s_{\text{vein}}^T$ (Fig. 5 (d)).

The vein particles and springs are connected following the hierarchical structure of the leaf’s unique venation patterns [RFL05]. In this paper, we mainly focus on leaves of dicotyledons, in which the veins originate from the stem of the leaf and branch repeatedly; these patterns are more interesting patterns than those of the parallel veins of monocotyledons. Murray’s law [Mur26] states that the radii of daughter veins $r_{di}$ can be obtained from the radius of the mother vein $r$ using the following relationship: $r^3 = \sum r_{di}^3$. Therefore, the main vein has the maximum radius and the radii of veins decrease following the venation hierarchy. In addition, because a leaf vein tends to taper to the tip, the vein particles along the same vein have different radii.

To represent the non-uniform distribution of vein radii following Murray’s law, we need to interpolate the quantities, such as mass of particles, elsewhere on a layer using the
weighted distance from the particles to the leaf base. To do this we first initialize the distance \( d_i = 0 \) of the leaf base particle and find the shortest distance \( d_i \) of particles from the leaf vein using the Bellman-Ford algorithm with different weights for vein and remaining surface springs, \( \omega_{\text{vein}} \) and \( \omega_{\text{surf}} \). The interpolation parameter \( \tau_i \) (0 ≤ \( \tau_i \) ≤ 1) for a particle \( p_i \) is then calculated using the following kernel function:

\[
\tau_i = \left( 1 - \frac{d_i}{d_{\text{max}}} \right)^D,
\]

where \( d_{\text{max}} \) is the maximum value in \( \{d_i\} \). Attaching smaller weights \( \omega_{\text{vein}} \) to the lengths of the vein springs in the Bellman-Ford algorithm and using higher value of \( D \) in the kernel function changes the dispersion of \( \tau_i \). As \( D \) increases, only particles close to the vein have large \( \tau_i \), as shown in Fig. 7(d).

Then, the initial mass of particles \( m_i^0 \) is linearly interpolated by \( \tau_i \).

\[
m_i^0 = m_{\text{min}}(1-\tau_i) + m_{\text{max}}\tau_i,
\]

where \( (m_{\text{min}}, m_{\text{max}}) \) is the range of mass.

4. Osmotic Water Flow

A leaf cell is bounded by a permeable membrane which allows water but not solutes in and out of the cell [KB95]. When the concentration of solutes increases as a result of transpiration or evaporation, water flows between cells to equalize concentrations. In our particle-spring system a cell is represented as a particle with an amount of water \( w_i \) and of solutes \( m_i \), and the permeable membrane between two cells is represented by the spring which connects the two particles.

In an equilibrium state, the concentration of solutes \( c_i = M_i/m_i \) and the water ratio \( \mu_i = w_i/m_i \) is the same in all particles (Fig. 8 (a)). As soon as a particle becomes dehydrated (Fig. 8 (b)), water starts moving up the concentration gradient until the particles return to an equilibrium (Fig. 8 (c)).

To simulate the flow of water through the network of veins and its diffusion to the edge of the leaf, the particle masses \( m_i^f \) are distributed initially using the interpolation parameter \( \tau_i \), within the range \( m_{\text{min}} \) and \( m_{\text{max}} \). The mass of Voronoi particles \( m_i^f \) is the average mass of the three particles of the corresponding triangle. Then, the water content \( w_i \) of all particles is determined by the same water ratio \( \mu_i^0 \) \( (w_i = \mu_i^0 m_i, \quad 0 \leq \mu_i^0 < 1) \). When the rate of water loss is approximately the same \( \Delta w_i \equiv -w_{\text{loss}}\Delta t \) at every particle, the concentration of solutes of particles with low mass increases quickly, and water flows into them.

We model osmotic water flow using an extension of the 1D diffusion method introduced by Desbrun et al. [DMSB99]. They proposed the umbrella operator \( L \) (Eq. 3) weighted by the spring length to solve the problem of non-uniform diffusion on irregular mesh. They also represent the implicit integration (Eq. 4) for stable simulation when the time step \( \Delta t \) and diffusion coefficient \( \lambda \) are large.

\[
L(w_i) = \frac{2}{E} \sum_{j \in N(i)} \frac{\Delta w_{ij}}{|l_{ij}|}, \quad \text{where} \quad E = \sum_{j \in N(i)} |l_{ij}|, \quad \text{Eq. 3}
\]

where \( \Delta w_{ij} = w_j - w_i \) and \( l_{ij} = |x_j - x_i| \).

\[
(1 - \lambda \Delta t L) w^{t+1} = (w^t - w_{\text{loss}} \Delta t I), \quad \text{Eq. 4}
\]

In general diffusions, the quantity of exchanged water between two particles is proportional to the difference in the mass of water in the two particles \( \Delta w_{ij} \) in Eq. 3. To model osmotic flow, \( \Delta w_{ij} \) must be the amount of water transferred because of the difference in concentration.

In order to equalize the concentration in two particles, an amount of water \( \Delta w_{ij} \) moves from \( p_j \) to \( p_i \):

\[
\frac{M_j}{M_j + w_j - \Delta w_{ij}} = \frac{M_i}{M_i + w_i + \Delta w_{ij}}, \quad \text{Eq. 5}
\]
If we reformulate Eq. 5 to express $\Delta w_{ij}$, we obtain

$$\Delta w_{ij} = \frac{M_i w_j - M_j w_i}{M_i + M_j} \quad (6)$$

Consequently, the Laplacian operator for osmosis is derived by substituting $\Delta w_{ij}$ in Eq. 3, as follows:

$$L^o(w_i) = \frac{2}{E} \sum_{j \in N(i)} \lambda_{ij} \frac{M_i w_j - M_j w_i}{|l_{ij}| (M_i + M_j)}. \quad (7)$$

To model the faster flow of water through veins, we weight the exchanged water between two particles $\Delta w_{ij}$ in the Laplace operator (Eq. 7) by replacing the diffusion coefficient $\lambda$ in Eq. 4 with $\lambda_{ij}$ to obtain:

$$L^o(w_i) = \frac{2}{E} \sum_{j \in N(i)} \lambda_{ij} \frac{M_i w_j - M_j w_i}{|l_{ij}| (M_i + M_j)}. \quad (8)$$

$$(1 - \Delta L^o) w^{n+1} = (w^n - w_{loss} \Delta t) \quad (9)$$

Since springs are not directional links ($\lambda_{ij} = \lambda_{ji}$), the matrix $(1 - \Delta L^o)$ is positive definite, and so the conjugate gradient method converges. The values of $\lambda_{ij}$ are obtained in the context of the vein structure using the averaged interpolation parameter of the two connected particles $(\tau_i + \tau_j)/2$ within the range $\lambda_{\text{min}}$ to $\lambda_{\text{max}}$.

As a result of the change of water mass $w_i$, the total mass $m_i$ has to be updated:

$$m_i^n = M_i + w_i^n \quad (10)$$

Fig. 9 compares results obtained from the osmotic model with those from a general diffusion model [DMSB99]. In (a), we see a triangular mesh structure filling a rectangle; (b) shows the initial mass of solutes $M_i$ in particles — those in the primary vein (white) are initialized to 1.0, those in the secondary veins (gray) are 0.55, and the remaining regions (black) are 0.1. $\lambda$ of a spring is interpolated by $M_j = (M_a + M_b)/2 - 0.1$ where $M_a$ and $M_b$ are the solutes masses of two particles such that $\lambda$ in the black region is zero: $\lambda = \lambda_{\text{min}} (1 - M_j) + \lambda_{\text{max}} M_j$.

In the initial state, all particles are completely desiccated, $w_i = 0$, and a constant flow of water is supplied to the leaf base (the red circle in (a)) during the simulation. In (c) we see how the general diffusion [DMSB99] causes water to spread out in all directions regardless of the mass of solutes until all particles have the same water content in the equilibrium state.
The experiment compares two methods to control the water capacity and the flow of water \( w_j \) by the distribution of solutes \( M_i \); (d) the general diffusion with varying diffusion coefficient \( \lambda \) between \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \) and (e) the osmotic water flow. In (d), \( \lambda = 0 \) for the springs in the black regions, preventing water flow into the black regions. However, \( \lambda \neq 0 \) cannot cause particles to have different water content as found in (e), because \( \lambda \) only affects the speed of the diffusion. On the contrary, in (e), the water content in the black regions is low \((< 1.0)\) and particles containing more solutes in the white regions accept more water than those in the gray regions. If we combine the osmotic flow water (e) with the different values of \( \lambda \) (d), we can simulate the faster flow of water along the white veins containing more water than other regions (f).

5. Leaf Deformation

When springs in the triangle-based double-layer structure shrink or expand, the structure bends or wrinkles so as to minimize the compressive or tensile stress. We consider the two types of bending of the leaf surface caused by differential strain in and out of the plane of the leaf [CRM04]. When the upper and lower surfaces of a leaf shrink by a different amount, the leaf curls up (Fig. 14 (a)). Drying leaves generally curl up, because the upper surface receives more sunlight directly and dries quickly. At the same time, non-uniform shrinkage in each layer produces in-plane stresses, and leaf surface buckles (Fig. 14 (b, c)). We therefore handle bending by shrinking or expanding the springs in the double-layer-structure. The realism of bending and wrinkling is enhanced by robust strain-limiting techniques [MHHR07].

5.1. Leaf Shrinkage by Dehydration

We regard that the mass of a spring is the average mass of two connected particles. If the density of the spring \( \rho \) is constant, then the volume \( v \) is linearly proportional to the mass of the spring. Then, the ratio of the volume of a spring with regard to its initial value is as follows:

\[
v_{\text{rel}} = \frac{(m_0^a + m_0^b) \rho}{(m_0^a + m_0^b) \rho} = \frac{m_a^0 + m_b^0}{m_a^0 + m_b^0}
\]

The target length of a spring is therefore adjusted to account for a change in particle mass as follows:

\[
l = l^0 \left( 1 - \sigma (1 - v_{\text{rel}})^{\alpha} \right),
\]

where \( \sigma \) is a user-defined parameter and \( l^0 \) is the rest length.

The user provides \( \sigma_{\text{vein}} \) to the shrinkage rate for veins, and \( \sigma_{\text{surf}} \) for the shrinkage rate on the surface. Large values of \( \sigma_{\text{vein}} \) are assigned to vein springs to create sharp creases along veins. As the masses of particles in the two layers decrease by water loss, the leaf model bends in the direction of the layer that loses more water. The thickness of the leaf can also decrease as the length of springs in the linked layer decreases. If the particle masses, \( m_a \) and \( m_b \), are under the threshold \( m_{\text{min}} \) or the spring length \( l_{\text{min}} \) is under the threshold \( l_{\text{min}} \), the spring stops shrinking.

Woolley [Woo73] shows that the change in volume of dehydrated leaf pieces is linearly proportional to the decrease of the relative water content; therefore, the changes in thickness and the decrease in area are non-linear by dimensional considerations. In our simulation, we use \( \alpha = 1 \) for simplicity and efficiency, but this results in the non-linear decrease in volume as shown in Fig. 10 (a). Instead of formulating a new equation, we assign \( \alpha \approx 1.3 \) to make the volume changes quite linearly dependent on the mass changes (Fig. 10 (b)).

5.2. Dynamics

Overly stretched or compressed springs produce excessive distortion of the shape of a rubber-like behavior. Compared to earlier techniques relying on internal forces, the constraint-based technique [MHHR07] places narrow limits on the deformation of springs and converges much more quickly.

In the position-based dynamics (PBD) approach [MHHR07], a spring with a target length \( l \) that connects the two particles \( p_a \) and \( p_b \) is subject to the strain-limiting constraint

\[
C(x_a, x_b) = |x_a - x_b| - l.
\]

In the PBD approach, the position of the endpoints are iteratively displaced in the direction opposite to the gradient of the constraint \( -\nabla C \) until the strains of all springs are under the user-defined threshold: \( |C/l| < \lambda_C \). The constraint on a spring induces the displacements \( \Delta x \) of its end points, and their velocities also change: \( \Delta v_i = \Delta x_i / \Delta t \).

The constraint becomes effective when particles are displaced by external forces or the target length of a spring is adjusted to model shrinkage or expansion. In our simulation, the constraint value increases as the spring shortens by dehydration. Then, we update the position and velocity of particles using the PBD to find the deformed shape to minimize the constraint.
5.3. Collision Detections

After solving the PBD system, we resolve self-collisions on the triangular mesh alone. When a particle on the triangle layer passes through the Voronoi layer as a result of a displacement introduced to correct a self-collision, the positions and velocities of Voronoi particles are also modified.

A Voronoi particle $p^V_i$ is supposed to maintain the thickness of the leaf by keeping a distance $\hat{h}_i$ from the corresponding triangle $F$ on the triangle layer. Thus, the position of a Voronoi particle can easily be updated as follows:

$$x_i^V = \frac{1}{3}(x_a^F + x_b^F + x_c^F) + h_in,$$  \hspace{1cm} (14)

where $x_a^F$, $x_b^F$, and $x_c^F$ are the positions of three particles of $F$, and $n$ is the normal to the plane of $F$. The new velocity of the Voronoi particle $v_i^V$ is the average of those of the three triangle particles $(v_a^F + v_b^F + v_c^F)/3$.

The triangle and Voronoi layers often entangle each other when a particle in one layer penetrates the surface of the other and gets stuck because of the action of the link springs (Fig. 11(b)). The creases produced by the mislocated particles may be considered as errors; but they also help to make wrinkles more complicated, as shown in Fig. 12 (b).

Therefore, the distance $\tilde{h}_i = (x_i^V - x_a^F) \cdot n$ is temporarily stored just before collision detection, and the positions of the Voronoi particles are restored after the effect of collisions has been resolved by using $h_i$ or $\tilde{h}_i$ in Eq. 14.

![Figure 11: (a) Two-layer structure at rest. (b) A particle on the Voronoi layer passes through its corresponding triangle and generates curves.](image)

![Figure 12: The generated wrinkles with (a) $h_i$ and (b) $\tilde{h}_i$.](image)

6. Results

We used a PC with an Intel Core i5 CPU running at 2.67GHz, with 16GB of RAM, and implemented our simulation program using C++. The time-consuming step in the simulation is solving the PBD systems in the deformation and it occupies 86\% of the total simulation times. The computational cost for the PBD mainly depends on the number of springs. For example, solving the PBD system for the maple model (Fig. 17) which contains 51K springs took 1157ms and the total computation time per frame was 1337ms.

The pseudo-code in Algorithm 1 summarizes the whole simulation we have explained. When one of the two layers is desiccated completely, we stop the simulation of the osmotic water flow and shrinkage of springs to prevent returning to the flat shape. If the length of a spring is shorter than the user-defined ratio, it also stops shrinking. Even if the leaf model finishes drying and shrinking, we can continue the simulation of the leaf being moved and deformed by external forces such as gravity or collisions with the environmental objects.

We cannot synchronize the simulation and real time, because a leaf dries over several hours or days. The time step could be arbitrarily determined by the user, but is restricted by the condition of each simulation. The PBD [MHHR07] and the implicit integration for diffusion [DMSB99] that we

Algorithm 1 The pseudo-code for the simulation.

1: for each frame $n$ do
2: $w^f \leftarrow$ osmoticWaterFlow($p^V_i$, $x_i^V$, $w_{max}$) (Eq. 9)  
3: $w^v \leftarrow$ osmoticWaterFlow($p^V_i$, $x_i^V$, $w_{max}$)  
4: for each particle $p^V_i \cup F$ do  
5: $m_i \leftarrow$ updateMass() (Eq. 10)  
6: end for  
7: for each spring $s_{ij}^U \cup L$ do  
8: $l_{ij} \leftarrow$ shrinkSpringLength() (Eq. 12)  
9: end for  
10: for each particle $p^V_i \cup F$ do  
11: $v_i \leftarrow v_i + g\Delta t$, \hspace{0.5cm} (g: gravity)  
12: $x_i \leftarrow x_i + v_i\Delta t$  
13: end for  
14: for each particle $p^V_i \cup F$ do  
15: $h_i \leftarrow (x_i^V - x_a^F) \cdot n$  
16: end for  
17: for each projection step $k$ do  
18: $\tilde{h}_i \leftarrow (x_i^V - x_a^F) \cdot n$  
19: $x_i^f, v_i^f \leftarrow$ PBD($x_i^f, C^f$)  
20: $x_i^v, v_i^v \leftarrow$ PBD($x_i^v, C^v$)  
21: $x_{ij}^U, v_{ij}^U \leftarrow$ PBD($x_{ij}^U, C_{ij}^U$)  
22: end for  
23: $x_i^f, v_i^f \leftarrow$ resolveCollisions($p^V_i, F$)  
24: for each particle $p^V_i \cup F$ do  
25: $x_i^v \leftarrow (x_i^f + x_a^F + x_b^F)/3 + \tilde{h}_i n$, \hspace{0.5cm} (or $h_i$)  
26: $v_i \leftarrow (v_a^F + v_b^F + v_c^F)/3$  
27: end for  
28: end for
Table 1: The computation times (ms) for each simulation step (The examples in Fig. 17).

<table>
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<tr>
<th></th>
<th>Maple</th>
<th>Plane Tree</th>
<th>Wisteria</th>
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<td>Total (ms)</td>
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<td>1602.73</td>
<td>1538.83</td>
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</table>

use are relatively stable with a large time step, but the collision detection limits the time step in our simulation.

All the simulation results are rendered with Mental Ray in 3ds Max. The photographs of leaves in the modeling process are used as leaf textures for rendering. To visualize the smooth color transition of drying leaves, we linearly blend each pixel color of the input leaf texture with the user-defined desiccated leaf texture depending on the relative water content.

We demonstrate the diversity of simulation results that can be obtained by adjusting the main parameters in each simulation step, and verify the realism of our technique by comparing our results with photographs of real leaves.

**Osmotic water flow.** The flow of water can be controlled by adjusting the parameters that determine the amount and distribution of water, which are \( \tau \), the diffusion coefficient \( \lambda \) and the boundary conditions of the diffusion. Fig. 16 shows leaves which have half become dehydrated at a constant rate. When \( \lambda = 0.0 \) (b), the relative water content \( \frac{w_o}{w_0} \) (bottom) is associated with the venation patterns, whereas when \( \lambda = 100.0 \) (c) the concentration of water has dropped across the entire leaf; therefore, the leaf shrinks and wrinkles form over its whole surface.

**Shape deformation by shrinkage.** The thickness \( h \) of the double-layer structure affects the fine details including wrinkles, curls and creases on the leaf surface; but it cannot be related to biology and must be chosen by the user. Fig. 13 shows wrinkles on surfaces of different thickness. As we would expect, the thinnest surface (c) has the finest wrinkles whereas the thickest surface (a) bends smoothly. Fig. 14 also compare the wrinkles of two circles with different thickness when they shrink non-uniformly.

The shrinkage rate of springs \( \sigma_{vein} \) on the Voronoi layer is used to generate sharp creases or folds along main veins. As \( \sigma_{vein} \) increases, the creases become prominent (Fig. 15 (b), (c)). When \( \sigma_{vein} \) is negative, the direction of the crease reverses (Fig. 15 (a)).

**Comparisons with real leaves.** We observed the changes that occurred in sample leaves as they dehydrated. The leaf surface, real photos and the corresponding vein structure are shown in Fig. 17. Tables 1 and 2 show the computation times and the parameters used in the simulation for the leaves.

The leaves of plane trees, wisteria and hibiscus show similar deformations in which veins become obtrusive and fine wrinkles appear on the leaf surface as the leaves shrink. The boundary region of the red Japanese maple leaf wrinkles or curls up, whereas the green Japanese maple generates sharp creases along the main veins rolling at the tip. These two maples uses the same particle-spring model, but produce dif-
different results with different textures and parameters. Drying ginkgo leaves do not distort much, but the boundaries of the leaves roll up. Because our technique does not entirely rely on the vein of a model, it can depict the drying of leaves without prominent veins.

Although we mainly focus on the leaves of dicotyledons, we can also simulate the deformation of leaves of monocotyledons, which have parallel veins branching at the leaf base and merging at the tip. The grass leaf model rolls from the tip as the dehydration proceeds.

### 7. Discussion

The venation patterns shown in the examples given are quite simple, because we manually modeled the auxiliary image (Fig. 3(b)) which identifies the vein structure. We can replace the manual process with automatic vein extraction to generate more complex venation patterns and to enhance the efficiency. The extraction of veins from the leaf silhouette [MMP03] is limited to the lobed shape and cannot guarantee the correspondence between the resulting vein skeletons and the venation patterns in the input texture. We believe that image processing [PHN08] techniques could extract more accurate venation patterns of the given textures.

Runions et al. [RFL∗05] generate realistic leaf venation patterns by the gradual development of veins based on the canalization hypothesis. This technique produces various types of venation patterns by adjusting parameters. By defining the sampled points following the synthesized veins as the vein particles in our modeling process, we can generate a diversity of input leaf models with different venation patterns.

Various effects such as cracks, insect attacks [PTMG08] and growing spots could be incorporated into the simulation to improve realism. Because leaves tend to become less elastic or rigid as they are dehydrated, we must treat them as less elastic or rigid bodies. Using the PBD [MHHR07], the stiffness coefficient $k$ ($0 \leq k \leq 1$) multiplied by the displacement $\Delta x$ or the threshold $\lambda_{C}$ for convergence affects the stiffness of the model (Section 5.2). We could therefore simulate the change in the stiffness of leaves by interpolating $k$ and $\lambda_{C}$ depending on the relative water content.

Simulation methods provide diverse results by changing parameters, but this requires much work by the user to determine the optimal values for the required results. Similarly, the proposed technique also takes time to adjust parameters by trial and errors, because the simulation is not in real time and there are many parameters to consider. To avoid this, measured data from experiments [XC11] could be a solution. Although we have tried to emulate the biology of real leaves as far as possible, many influencing factors have been neglected for the sake of manageable controls and reasonable computation times.

The modeling of deformation by means of inhomogeneous volume changes could be extended to model other botanic processes, such as the growth, wilting and drying of leaves or flowers [IYKI08]. Osmotic water flow changes the concentration of solutes towards an equilibrium state. This suggests that our technique might be applied to other processes based on diffusions.

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### Table 2: The parameter values for the examples in Fig. 17.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Maple (green)</th>
<th>Maple (red)</th>
<th>Plane Tree</th>
<th>Wisteria</th>
<th>Grass</th>
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<td>$\Delta t$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td>$g$</td>
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<td></td>
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<td></td>
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<tr>
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<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D$</td>
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<td>2</td>
<td>1</td>
<td>2</td>
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<td>0.06</td>
<td>0.01</td>
<td>0.02</td>
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<tr>
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<td>0.01</td>
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<td>0.3</td>
<td>10.0</td>
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---

**Figure 16:** Changes in water content when a leaf is semi-dehydrated. When $\lambda$ is small, the variation in the water content increases, because the regions away from veins cannot be rehydrated quickly. (a) A fully hydrated leaf and dehydrated leaves with (b) $\lambda = 0.0$ and (c) $\lambda = 100.0$. The upper images show leaf color, water content is shown in the middle images, and relative water content below.
8. Conclusions

We have simulated the changes in morphology found in drying leaves. Using triangle-based double-layer structure allowed us to model the fine wrinkles caused by inhomogeneous shrinkage due to the water loss. The osmotic water flow simulates the internal water flow through leaves so that dehydrated regions of drying leaves advance forward the veins.

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Figure 17: The drying of different types of leaf: Japanese maple (red), Japanese maple (green), plane tree, hibiscus, ginkgo, wisteria and grass leaf (from top to bottom images). (a) The animation of drying leaves. (b) The results of simulated leaf models and (c) photographs of leaves. (d) The vein structures.